RAMAKRISHNA MISSION VIDYAMANDIRA
(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SIXTH SEMESTER EXAMINATION, MAY 2025
THIRD YEAR [BATCH 2022-25]
PHYSICS (HONOURS)
Paper : DSE 3

Date : 10/05/2025
Time : 11 am – 1 pm
Full Marks : 50

(General Theory of Relativity and Astrophysics)

Answer **any five** questions: [5×10]

1. a) What do you mean by celestial coordinate system? How do you specify the position of a star in celestial coordinates? Star Electra have the coordinates $(\alpha, \delta) = (3^h 44^m 53^s, 24^\circ 06' 47'')$ and star Taygeta have coordinates $(3^h 45^m 13^s, 24^\circ 28' 01'')$

   (i) What is the angular separation between these two stars.

   (ii) If both the stars are at a distance of 405 light years from Earth, how far the stars are from each other.

   b) What is UBV system, how can you estimate about the surface temperature of a star using UBV system?

   c) Redshift parameter z is defined as $z = \frac{\lambda_{obs} - \lambda_{rest}}{\lambda_{rest}}$. Derive an expression for redshift parameter z in terms of receding velocity of a relativistic particle. In its rest frame, quasar Q2203+29 produces a hydrogen emission line of wavelength 121.6 nm. Astronomers on Earth measure a wavelength of 656.8 nm for this line. Determine the redshift parameter and the apparent speed of recession for this quasar. [(1+1+2+1)+(1+1)+(1+1+1)]

2. a) Write down the radiative transfer equation in presence of matter along the ray path. Find the general solution of the above radiative transfer equation as a function of optical depth. After making necessary approximations now explain why stars emits like a blackbody.

   b) Why hydrogen lines are found in stars of intermediate temperature?

   c) Consider a star of mass $M = 10M_\odot$ and radius $= 0.7R_\odot$, composed entirely of fully ionized ^{12}C. Its core temperature is $T_c = 6 \times 10^8$ K .

   (i) If the luminosity of the star is L= $10^7\ L_\odot$, then what is the effective surface temperature?

   (ii) Suppose the star produces energy via the reaction

   $^{12}\text{C} \quad + \quad ^{12}\text{C} \quad \rightarrow \quad ^{24}\text{Mg}$

   The atomic weight of ^{12}C is 12, and that of ^{24}Mg is 23.985. What fraction of the star's mass can be converted into thermal energy?

   (iii) How much time does it take for the star to use up 10% of its carbon? [(1+2+2)+2+(1+1+1)]

3. a) Let X, Y and Z be the mass fraction of hydrogen, helium and other heavier elements inside a normal star. Show that the equation of state of this star is $P = \frac{k_B}{\mu m_H}\rho T$, where $\mu = \left(2X + \frac{3}{4}Y + \frac{1}{2}Z\right)^{-1}$ is the mean molecular weight.

   b) The pressure inside a normal star is given by $P = P_g + P_{rad}$, where $P_g = \frac{k_B}{\mu m_H}\rho T$ is the kinetic gas pressure and $P_{rad} = \frac{1}{3}\sigma T^4$ is the radiation pressure, where $\sigma = 5.67 \times 10^{-8}$ Wm^{-2}K^{-4} is the Stefan-Boltzmann constant. Show that at $0.9R_\odot$ of the Sun where $X = 0.71$, $Y = 0.27$ and

$Z = 0.02$ mean density $\rho = 1400$ kgm^{-3} and internal temperature is about 10^5 K, the kinetic gas pressure dominates. In the central region $X = 0.34$, $Y = 0.64$, $Z = 0.02$, central density $\rho_c = 1.5 \times 10^5$ kgm^{-3} and central temperature is $T_c = 1.5 \times 10^7$K, which pressure dominates? [Hint: Boltzmann constant $k_B = 1.38 \times 10^{-23}$ m^2 kg s^{-2} K^{-1} and mass of hydrogen $m_H = 1.67 \times 10^{-27}$ kg.]

c) What are sunspots? How does they appear on the surface of the Sun? Calculate the magnetic pressure in the center of the umbra of a large sunspot. Assume that the magnetic field strength is 0.2 T. [3+(2+2)+(1+1+1)]

4. a) Explain the features of Hertzsprung-Russel (H-R) diagram. Draw the diagram and locate the position of main sequence, while dwarf and red giant stars in the diagram.

b) If Sun's energy is only due to gravitational collapse of gas and mass of Sun is 1.99×10^{30} kg and radius 6.98×10^{8} m. Use Virial theorem to estimate the life span of Sun using Kelvin-Helmholtz law. Assume the Sun raidates energy at 3.826×10^{26} J/s.

c) What do you mean by Jean's instability? Estimate an expression for Jean's mass and Jean's radius. For a typical molecular cloud at 50K and number density 5×10^{8} m^{-3} is contracted to form a protostar of typical density 8.4×10^{-19} kg/m^3. Find the value of Jean's mass and Jean's radius.

[(1+2)+2+(1+2+2)]

5. a) Define a four vector? Generate the spherical basis vectors in terms of the Cartesian basis vectors $e_1 = \hat{x}, e_2 = \hat{y}, e_3 = \hat{z}$, using the transformation rule $e'_a = \frac{\partial x^i}{\partial x'^a} e_i$. [Hint: Parametric equations for spherical polar coordinates: $x = r \sin\theta \cos\varphi$, $y = r \sin\theta \sin\varphi, z = r\cos\theta$.]

b) In Minkowski spacetime the electromagnetic field tensor is defined as $F_{ij} = \partial_i A_j - \partial_j A_i$, where $A_i \equiv (-\varphi, \vec{A})$ is the four potential of electromagnetic field. Using the transformation rule of a four-tensor of rank two, determine how the components of electric and magnetic field transforms under Lorentz transformation. [(1+3)+6]

6. a) Using equivalence principle determine how the clocks at different gravitational potential runs at different rate.

b) Using the relations $\Gamma^k_{ij} = \frac{1}{2} g^{kl}(-\partial_l g_{ij} + \partial_i g_{lj} + \partial_j g_{il})$ and $\partial_a g = g g^{ik} \partial_a g_{ik}$, where $g = \det g_{ab}$, show that (i) $\Gamma^a_{ba} = \partial_b\left(\ln\sqrt{-g}\right)$ and (ii) $g^{bc}\Gamma^a_{bc} = -\frac{1}{\sqrt{-g}}\partial_b\left(\sqrt{-g}g^{ab}\right)$. [5+(2+3)]

7. a) Using null geodesic in Schwarzschild metric

$$ds^2 = -\left(1-2\frac{GM}{r}\right)dt^2 + \left(1-2\frac{GM}{r}\right)^{-1} dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right),$$

drive an expression for effective potential. What are the different possible trajectories of light rays in Schwarzschild space time.

b) Determine all the nonzero Christoffel symbols for the wormhole geometry

$$ds^2 = -dt^2 + dr^2 + (b^2 + r^2)(d\theta^2 + \sin^2\theta d\varphi^2).$$ [(2+2)+6]

8. Solve the vacuum Einstein's field equation for static spherically symmetric metric given by

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2).$$ [10]

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(Condensed Matter Physics)

Answer **any five** questions : [5×10]

1. a) What is a reciprocal lattice ? Show that the reciprocal lattice corresponding to a FCC lattice is a BCC lattice.

   b) (i)What are the Brillouin zones ? Discuss it from the Bragg's diffraction condition.

   (ii) Calculate the volume of first Brillouin zone of a bcc lattice.

   c) What is atomic form factor ? For the hydrogen atom in its ground state, the number density is $n(r) = (\pi a_0^3)^{-1}\exp[-2r/a_0]$, where a_0 is the Bohr radius. Show that the form factor is $f_G = 16/(4 + G^2a_0^2)$, where $G$is the reciprocal lattice vector. [(1+2)+(2+2)+(1+2)]

2. a) Show that kinetic energy of a three dimensional gas of N free electrons at $T = 0$is $U_0 = \frac{3}{5}NE_F$, where $E_F$is the Fermi energy.

   b) Derive the expression for electrical conductivity on the basis of quantum free electron theory.

   c) What is the origin of piezo-electric effect? Mention one application of piezoelectric phonomenon. [4+3+(2+1)]

3. a) Consider a metal described by the free electron model subject to an AC electrical field$E(t) = \Re[E(\omega)e^{-i\omega t}]$at finite frequency ω. Derive an expression of frequency dependent conductivity. Discuss it behaviour at very low and high frequency.

   b) What are excitons? What are different types of excitons? Distinguish between them. [(3+1+1)(2+1+2)]

4. a) Show that number of orbitals in an energy band is 2N, where N is the number of primitive cells.

   b) An electron is confined to a one dimensional box of length 5Å. If the electron makes a transition from first excited state to the ground state, calculate the frequency of the emitted photon. [Given: the mass of electron = 9.31×10^{-31} kg]

   c) Define density of states (DOS). Obtain the expression of DOS for bulk and 2D systems. [2+2+(1+5)]

5. a) Show that for an intrinsic semiconductor the Fermi energy μ (measured from the top of the valance band) is given by, $\mu = \frac{1}{2}E_g + \frac{3}{4}k_BT\ln\left(\frac{m_h}{m_e}\right)$, where $E_g$is the enrgy gap.

   b) Show that the Hall coefficient of a solid, $R_H = \frac{1}{e}\frac{p-nb^2}{(p+nb)^2}$, where $b = \frac{\mu_e}{\mu_h}$ is the mobility ratio, n and p are electron and hole concentrations.

   c) In a band structure calculation, the dispersion relation is found to be $\epsilon_k = \beta(\cos k_xa + \cos k_ya + \cos k_za)$, where β is a constant and a is the lattice constant. Calculate the effective mass at the boundary of the first Brillouin zone. [4+4+2]

6. a) Consider a pair of electron in spin singlet state with total momentum equal to zero. The pair is in a metal just above the Fermi sea of energy E_F and density of states N. Show that a small attractive potential leads to the formation of Cooper pair. Assume that the potential only affects eletrons occupying state in a small energy shell ΔE above the Fermi sea.

b) 'The perfect diamagnetism and zero resistivity of a superconductor are two mutually exclusive properties' - Explain.

c) Estimate the London penetration depth for 'tin' (density 7300 kg m^{-3}). Given the atomic weight = 118.7 u, T_C = 3.7 K and effective mass of electron (m^*) = 1.9 m_e (symbols have their usual meanings). [5+2+3]

7. a) Show that the total magnetic flux passes through a superconducitng ring may assume quantized value.

b) Consider a Josephson junction i.e., two identical superconductors are separated by an thin insulatiing layer. Show that current density across the junction is $J = J_0 \sin\delta$, where $\delta$is the superconducting phase diferrence.

c) Show that quantum size effect becomes observable in the condition

$\Delta x \leq \frac{\hbar}{\sqrt{mK_B T}}$ (symbols have their usual meaning.) [4+4+2]

8. a) A paramagnetic atom having permanent moment $\vec{\mu}$ with a given resultant quantum number $\vec{J}$ is placed in a magnetic field $\vec{B}$. Obtain an expression of magnetization as a function of $\vec{B}$ and temperature T. Hence obtain Curie's law in the appropriate limit.

b) Show that at low temperature, the paramagnetic susceptibility of a Fermi gas of conduction electrons is independent of temperature. [(4+2)+4]

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